**Entrance Exam for Masters Course**
**October 2012 / April 2013 Entry**
**Graduate Program on Environmental Sciences**
**Graduate School of Arts and Sciences, The University of Tokyo**

# Specialized Subjects

August 28, 2012　13:00 － 16:00

Do not open the exam question book until permission is given. In the meantime, read the following directions carefully.

1. This question book is for applicants to the Masters' Graduate Program on Environmental Sciences.
2. This question book consists of 20 pages. If you find any pages missing or out of order, or any unclear letters, raise your hand to inform the proctor.
3. Choose 3 questions to answer from Question 1 through Question 18.
4. 3 answer sheets are provided. Use one answer sheet per question. You can use both sides of the sheets.
5. Write the question numbers you have chosen to answer, subject names, your name and applicant number in the spaces at the top of each answer sheet. Please follow the example below.　Please note that answer sheets without name and applicant number will be nullified.

Example

| Question Number | Subject Name | Name | Applicant Number |
|---|---|---|---|
| Question 14 | **Information Sciences** (2) | ○ ○ ○ ○ | ○○○○○○ |

6. Answer all questions in English.
7. The last 3 pages of this book are scrap paper. They can be detached from the book for use.
8. You are not allowed to leave the room after the start of the examination until the examination is complete.
9. You must return the question book, the answer sheets and the scrap paper.
10. Fill in your applicant number and name in the spaces provided below.

| | |
|---|---|
| Applicant Number | |
| Name | |

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## Contents

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## Question 1 Mathematics (1)

(1) For a square of side 80 cm, prove that, if 65 points are placed inside the square, then at least two points must be separated by a distance of less than 15 cm.

[Hint] Since the diagonal line of a square of side 10 cm has a length of about 14.1 cm, the distance between any two points in this square is less than 15 cm.

(2) Answer the following questions.

(a) Prove that the sequence 7, 77, 777, 7777, ... contains a pair of numbers that have the same remainder when divided by 2013.

(b) Prove that the sequence 7, 77, 777, 7777, ... contains a number which is divisible by 2013.

## Question 2 Mathematics (2)

For real numbers $x$, $y$, and $z$, define a $3\times3$ matrix $A$ as

$$A=\begin{pmatrix} x & y & z \\ y & z & x \\ z & x & y \end{pmatrix}.$$

Answer the following questions.

(1) Define a vector $\boldsymbol{u}_0$ as

$$\boldsymbol{u}_0=\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Show that $\boldsymbol{u}_0$ is an eigenvector of $A$, and give the corresponding eigenvalue $\lambda_0$.

(2) Let $\lambda_1$ and $\lambda_2$ be the eigenvalues of $A$ other than $\lambda_0$. Show that $\lambda_1+\lambda_2=0$.

(3) Determine $\lambda_1$ and $\lambda_2$ by using the relationship in question (2).

(4) Let $\omega=\exp(2\pi i/3)$, where $i=\sqrt{-1}$. Define vectors $\boldsymbol{v}$ and $\overline{\boldsymbol{v}}$ as

$$\boldsymbol{v}=\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ \omega \\ \omega^2 \end{pmatrix} \quad \text{and} \quad \overline{\boldsymbol{v}}=\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ \omega^2 \\ \omega \end{pmatrix}.$$

Represent vectors $A\boldsymbol{v}$ and $A\overline{\boldsymbol{v}}$ as linear combinations of $\boldsymbol{u}_0$, $\boldsymbol{v}$, and $\overline{\boldsymbol{v}}$.

(5) Assuming that $x+y\omega+z\omega^2\neq0$, represent the eigenvectors corresponding to the eigenvalues $\lambda_1$ and $\lambda_2$ by using $\boldsymbol{v}$ and $\overline{\boldsymbol{v}}$.

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Question 3 Physics (1)

**I.** A particle of mass $m$ is suspended from a fixed point O by a light inextensible string of length $l$, as shown in Figure 1. The pendulum moves in the vertical plane under uniform gravity. The angle between the string and the vertial line is $\theta$.
(1) Find the equation of motion for the pendulum.
(2) Find the kinetic and potential energies of the system. Show the sum of the kinetic and potential energies remains constant in the motion.
(3) Show that the motion of the pendulum can be approximated by simple harmonic motion when the pendulum undergoes oscillation of small amplitude. Find the period of the oscillations.

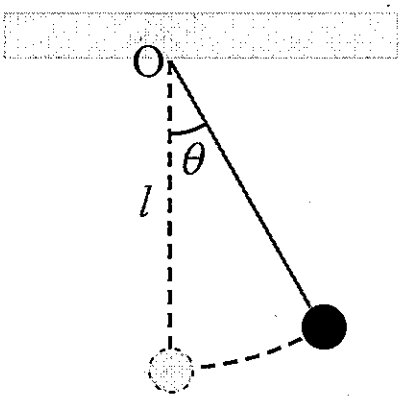


Figure 1:

**II.** Figure 2 shows two containers and a pipe connecting them. The cross sections of the both containers are $A$, and that of the pipe is $a$. The length of the pipe is $L$. The containers are partially filled with water, and a plate is floated over the surface of the water in the left-side container. The whole system is under uniform gravity. The water is incompressible and its density is $\rho$. The weight and thickness of the plate is negligible.
(4) A downward force $F$ was exerted on the plate in the left-side container and the water level of the right-side container rose from the original equilibrium level($y = 0$), and stayed there while the force was applied. Find the height ($y_0$) of the water level relative to the original level.
(5) When the force was suddenly removed, the water level started oscillating. Find the mechanical energy (the sum of the kinetic energy and the potential energy) of the water. The flow of the water $v$ in the pipe is uniform, and the flow in the container is negligible.
(6) Assuming the mechanical energy is conserved, find the period of oscillations.
(7) When there is friction between the water and the surface of the pipe, the mechanical energy is not conserved. Assuming the pressure drops $2\gamma\rho v$ per unit length ($\gamma$ is a constant), find the loss of energy per unit time. Find the condition under which the water level is damped toward the equibrium position without oscillations.

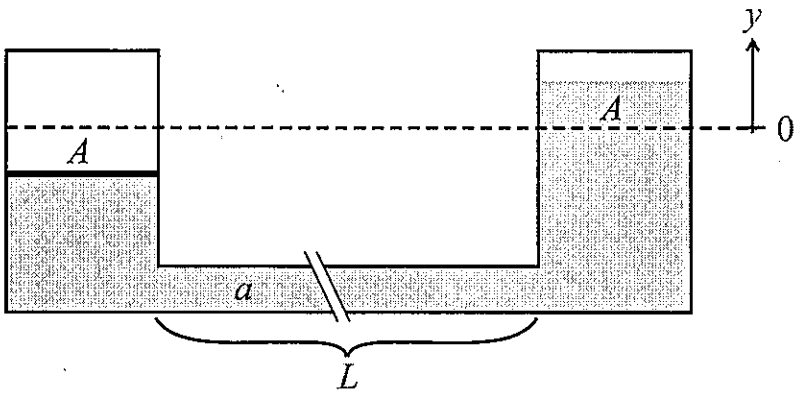


Figure 2:

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Question 4 Physics (2)

Figure 1 shows a rod of magnetic material of length $l$, which has a uniform magnetization $\boldsymbol{M}$ parallel to $\hat{z}$, and whose center is at the origin O. The rod is producing a magnetic field (magnetic flux density) $\boldsymbol{B}$. An atom is placed on the $z$-axis ($z > l/2$). The atom has a magnetic dipole moment $\boldsymbol{m}$, and the direction of $\boldsymbol{m}$ is parallel to $\hat{z}$. Answer the following questions. (Use symbol $\mu_0$ for the permeability of free space.)
(Hint: the magnetic moment $\boldsymbol{m}$ can be represented by a current loop as shown in Figure 2. Then $\boldsymbol{m} = IS\boldsymbol{n}$, where $I$ is current, $S$ is the area enclosed by the current, and $\boldsymbol{n}$ is the normal vector of the area.)

(1) What is the direction of the force acting on the atom?
(2) Let us consider $\boldsymbol{m}$ as a circular loop of current $I$ whose radius is $a$. Find the value of $I$.
(3) Choose the cylindrical coordinates $(r,\phi,z)$ with the $z$-axis along $\boldsymbol{M}$. Write the force acting on the atom using $B_a$, which is the $r$-component of $\boldsymbol{B}$ at the place where the circular current flows.
(4) Labelling the partial derivative of $B_z$ (the $z$-component of $\boldsymbol{B}$) at the position of the atom along the $z$-axis as $B'_z$, write the force using $B'_z$ and $m$. (Hint: use $\nabla \cdot \boldsymbol{B} = 0$)

A magnetic material which has a uniform magnetization $\boldsymbol{M}$ inside can be represented by the bound current on the surface of the material, as shown in Figure 3.

(5) Find the magnitude of the bound current per unit length along the $z$-direction.
(6) The length of the rod is much larger than the radius of the rod. Plot $B_z$ ($z$-component of the magnetic field $\boldsymbol{B}$) on $z$-axis as a function of $z$, with values at $z = 0$ and $z = \pm\frac{l}{2}$.

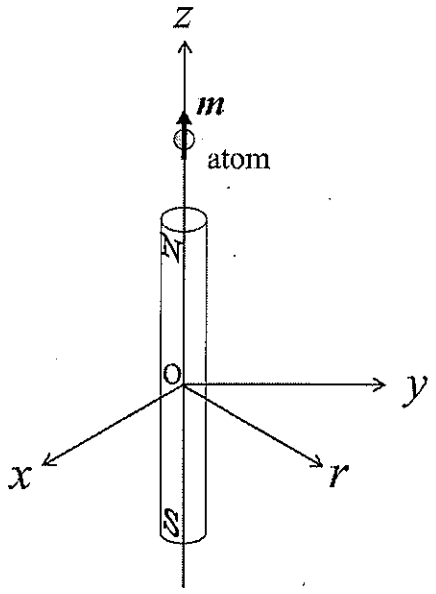

Figure 1

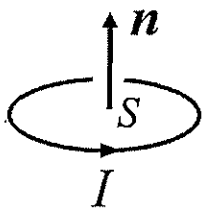

Figure 2

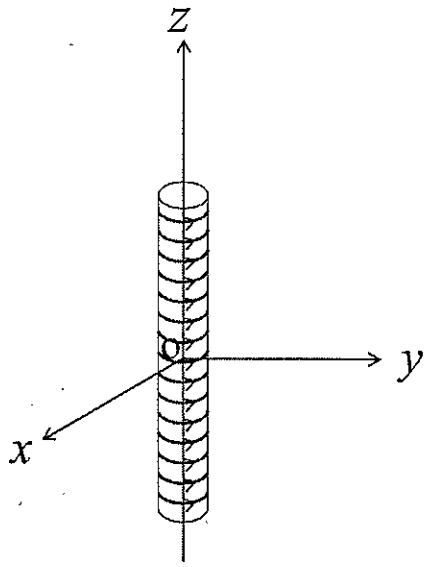

Figure 3

Question 5 Chemistry (1) - 1

**Answer all parts of all problems of I – III.**

I. Answer the following questions (1)~(8).

If necessary, use the following values of the physical constants: speed of light $c = 3.0 \times 10^8$ m s$^{-1}$, elementary charge $e = 1.6 \times 10^{-19}$ C and Planck constant $h = 6.6 \times 10^{-34}$ J s. The principal quantum number of an atom is expressed as $n$.

(1) Write the degeneracy of the energy state for $n = 3$ of a hydrogen atom, counting the combinations of the orbital angular momentum quantum number, $l$, and the magnetic quantum number, $m$.

(2) The orbital energy of the hydrogen atom is given by

$$E_n = -A/n^2$$

where $A$ is a constant value, and the ionization energy of the hydrogen atom is 13.6 eV. Calculate the orbital energy of the hydrogen atom for $n = 2$.

(3) The Lyman α line corresponds to radiation emitted upon transition from the $n = 2$ to the $n = 1$ state of the hydrogen atom. Calculate the wavelength of the Lyman α line in meters.

(4) Energies of the 1s, 2s and 2p orbitals of the carbon atom and the oxygen atom are given in Table 1. Explain why the energy of the 1s orbital of the oxygen atom is lower than that of the carbon atom.

(5) The 2s state and the 2p states are degenerate in the hydrogen atom. In contrast, as shown in Table 1 for multielectron atoms, the energy of the 2s orbital is different from the energy of the 2p orbital. Explain why the energy levels are split in multielectron atoms.

(6) An energy diagram of molecular orbitals for the oxygen molecule is schematically shown in Figure 1. Duplicate the energy diagram on the answer sheet, and draw the electronic configuration of an oxygen molecule in its ground state by filling the orbitals with electrons using ↑ and ↓.

(7) Calculate bond orders of $O_2$ and $O_2^+$ based on molecular orbital theory.

(8) Molecular orbitals, $1\sigma_g$ and $1\sigma_u$ of an oxygen molecule are considered to be composed of 1s atomic orbitals of two oxygen atoms, $\phi_{1s}^A$ and $\phi_{1s}^B$. Write the $1\sigma_g$ and $1\sigma_u$ orbitals using $\phi_{1s}^A$ and $\phi_{1s}^B$.

Table 1. Orbital energies of a carbon atom and an oxygen atom

| | Carbon atom (eV) | Oxygen atom (eV) |
|---|---|---|
| 1s | -284 | -543 |
| 2s | -19.2 | -33.9 |
| 2p | -11.3 | -13.6 |

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Question 5 Chemistry (1) - 2

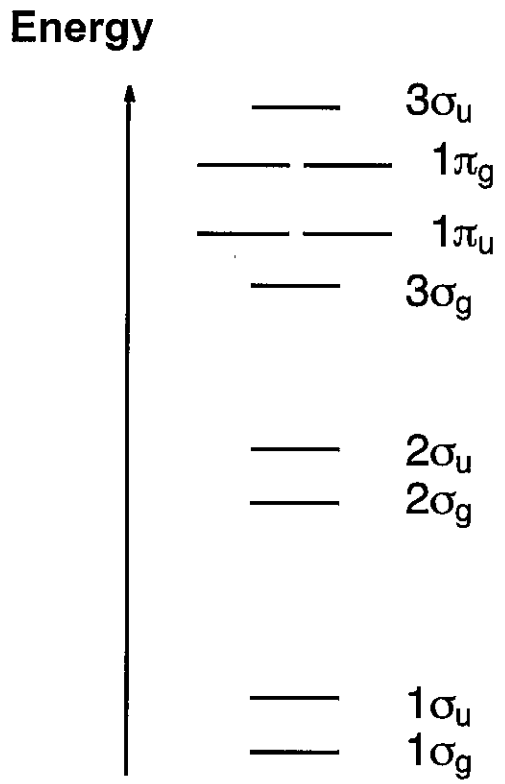



Figure 1. Energy diagram of molecular orbitals of an oxygen molecule.

II. Draw a Lewis structure for $XeF_4$ and predict its molecular structure using the valence-shell electron-pair repulsion (VSEPR) model. Rationalize your answer.

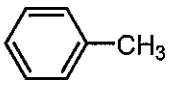
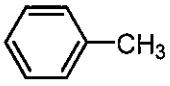
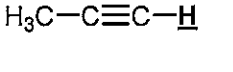
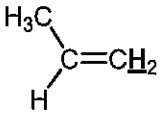
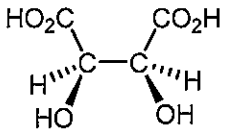
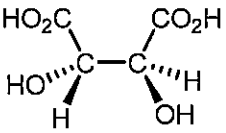
III. Which is the strongest acid among the following oxoacids? Rationalize your answer.

(A) HClO, (B) $HClO_2$, (C) $HClO_3$, (D) $HClO_4$

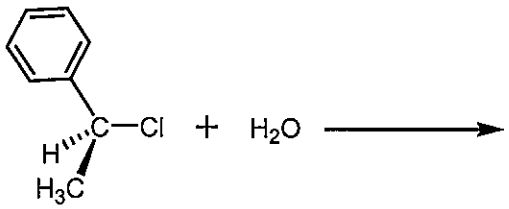
## Question 6 Chemistry (2) - 1

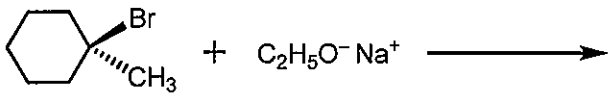
**Answer all parts of all problems of I – IV.**

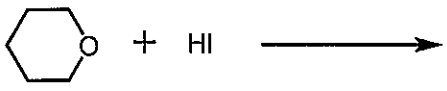
I. Which compound (A or B) is correct with respect to the subsequent statements (1) – (4)? Provide a reason for your decision.

| | A | B | Statement |
|---|---|---|---|
| (1) |  |  | Has a higher boiling point. |
| (2) |  |  | Has a higher melting point. |
| (3) |  |  | **H** is more acidic. |
| (4) |  |  | Not optically active. |

II. Determine the product(s) of the following reactions:

(1) 

(2) 

(3) 

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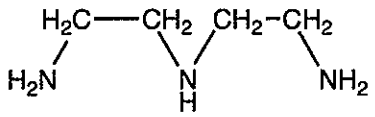
Question 6 Chemistry (2) - 2

III. Give the ground state electronic configuration of d electrons in the high spin octahedral complex $[Mn(H_2O)_6]^{2+}$ ($d^5$).

IV. Draw all structures of $Co^{3+}$ complexes with two tridentate ligands **A** (dien: diethylenetriamine, Figure 1), as schematically shown in Figure 2. For enantiomers, indicate the structures as a pair of mirror images.




**A**

Figure 1. The chemical structure of dien.

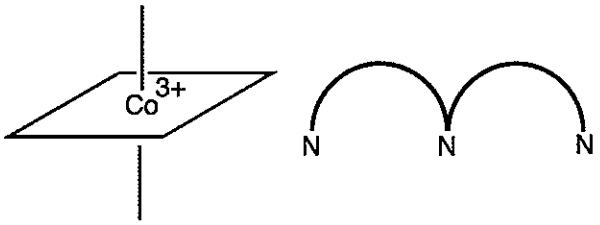



Figure 2. Schematic representation of an octahedral $Co^{3+}$ complex and dien.

## Question 7 Biology (1)

**Read the following text and answer the accompanying questions.**

The concept of genetic information flowing in one direction from DNA to mRNA to proteins is called the "$_{a}$central dogma" of molecular biology. This concept is a basic principle common to all organisms - both prokaryotes and eukaryotes - including bacteria and humans. mRNA synthesis means the $_{b}$transcription of the genetic information in DNA (the base sequence) to the base sequence of RNA, while protein synthesis refers to the $_{c}$translation of information in one language (the sequence of mRNA) into that of another (the amino acid sequence).

**Questions**

1. Underlined text a: The "central dogma", is an important principle of biology, but exceptions can be found. Briefly explain an exceptional mechanism and provide an example.

2. In the "RNA world" hypothesis, RNA is thought to be precedent to DNA and protein during the evolution of life on Earth. What are the special characteristics of RNA that make it an evolutionary precursor?

3. Underlined text b: In the transcription of eukaryotic mRNAs, which type of RNA polymerase is involved?

4. Underlined text c: For general secretory proteins:
   A) Which organelle does translation employ?
   B) Briefly explain how the proteins are processed in the cell after translation.

5. Outline the process that occurs during mRNA synthesis in eukaryotes between the transcription and completion of mRNA.

6. Which of the following is NOT a feature of eukaryotic gene expression?
   A) polycistronic mRNAs are very rare
   B) many genes are interrupted by noncoding DNA sequences called introns
   C) mRNA synthesis and protein synthesis are coupled
   D) multiple copies of nuclear genes and pseudogenes can occur

7. In eukaryotes, many types of protein can be synthesized from only one gene and that gene can function as multiple genes.
   A) What is the name of this mechanism?
   B) Briefly outline this mechanism.

8. In eukaryotic multicellular organisms, each differentiated cell type has a unique gene expression pattern - a characteristic that is preserved even after cell division. Although it may appear that these cells have different genes and pass them on to their progeny cells, the genes are in fact the same.
   A) What is the name of the gene-level change behind these phenomena?
   B) Describe the mechanism that causes such changes.

Question 8 Biology (2)

Answer the following four questions.

【1】 Select three of the following items and explain each in approximately 4 lines.

(1) The relationship between "ecosystem function" and "ecosystem service"
(2) Phenotypic plasticity of morphological traits
(3) Competition and choice in sexual selection
(4) Effects of dispersal in a meta-community
(5) Gap dynamics in forests
(6) The difference in the light photosynthetic curve and the light compensation point of shade leaves and sun leaves

【2】 Select two of the following items and explain each from an evolutionary biological point of view, in approximately 6 lines and using an actual example in each case.

(1) Adaptive radiation
(2) Sympatric speciation
(3) Convergent evolution
(4) Frequency-dependent natural selection

【3】 Describe the Lotka-Volterra prey-predator equations and explain how population density oscillates, in approximately 6 lines. You may use figures in your explanation.

【4】 Human activities such as overloaded fertilizers in farm land and domestic wastewater often cause eutrophication with excess nitrogen and phosphate in aquatic ecosystems. What does the eutrophication give rise to in aquatic ecosystems and communities and how does it do so? Give separate explanations for the cases of both lakes and coastal oceans using approximately 6 lines in each case.

Question 9 Health Sciences (1)

Over the past several decades, there has been an explosive increase in the prevalence of lifestyle-related diseases, including type2 diabetes mellitus, hyperlipidemia, hypertension and atherosclerosis, comprising the so-called metabolic syndrome. Evidence has been accumulating that calorie restriction (reduced daily energy intake) and regular exercise are both effective in preventing these diseases.

Briefly describe the mechanisms by which calorie restriction and exercise prevent lifestyle-related diseases, respectively, and point out the similarities and differences in their effects on metabolic functions.

Question 10 Health Sciences (2)

It is well known that high-intensity resistance training causes increases in muscle strength and size. A 20-year-old male started resistance training for the first time. He completed 3 months (3 days per week) of unilateral knee extension training {80% of 1 RM (repetition maximum) x 10 repetitions x 5 sets}.

How do muscle strength and size (cross-sectional area) change in the trained knee extensor muscles during the period of training? Describe the time course for changes in muscle strength and cross-sectional area separately.

What changes can be expected to occur in muscle strength and the cross-sectional area of knee extensor muscles in the opposite limb (untrained limb) after 3 months of training?

## Question 11 Earth Sciences (1)

Answer the following questions concerning the general circulation in the atmosphere and oceans:

(1) On a spatial scale of 1000 km or more, how is the direction of wind related to isobars on the ground? Answer the question for all of the following cases: (a) on the equator, (b) at high latitude in the northern hemisphere, and (c) at high latitude in the southern hemisphere. You may use drawings, as necessary. Also, explain why the relationship between wind direction and isobars you identified arises.

(2) On the Earth, why does an easterly wind, called a trade wind, blow in tropical regions along the equator, and why does a westerly wind blow at mid-latitude? Include the following words in your answer:

Hadley circulation, intertropical convergence zone, subtropical high

You may use drawings, as necessary.

(3) What ocean currents do the westerly wind and the easterly wind you discussed in (2) induce in the surface water of the Pacific Ocean and the Atlantic Ocean? You may use drawings, as necessary.

## Question 12 Earth Sciences (2)

Consider plate tectonics, and variations of magmatisms and compositions of the magmas on the earth.
(1) Plate boundaries can be subdivided into three types. Explain the three types, with representative examples.

(2) Give three types of geological setting of magmatisms on the earth based on the concept of plate tectonics.

(3) Explain compositions and origins of the magmas formed at the magmatisms.

(4) Figures 1a and 1b show phase relations of forsterite-enstatite-$SiO_2$ binary systems at 1.5 and 2.5 GPa, respectively.

(a) Assuming that a starting material has composition "A", show compositional changes of the formed melts from their solidus temperatures to 2000 °C for 1.5 and 2.5 GPa using the figures, respectively. If necessary, draw the figures on your answer sheet.

(b) Figures 1a and 1b show that the compositions of the solidus melts at 1.5 and 2.5 GPa are different to each other. Describe the types of magmas that arise under these two conditions.

(c) Assuming that melts have composition "A", show compositional changes of the melts from their liquidus to solidus temperatures for equilibrium and fractional crystallization using both the figures at 1.5 and 2.5 GPa, respectively. If necessary, draw the figures on your answer sheet.

(d) Write mineral assemblages after solidification for equilibrium and fractional crystallization at 1.5 and 2.5 GPa, respectively.

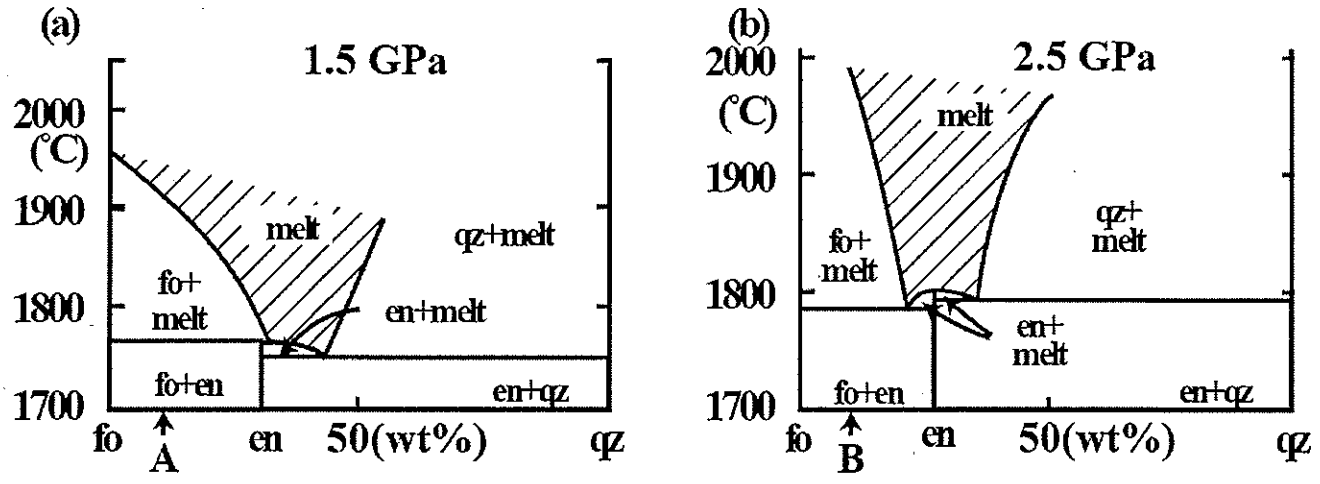



Figure 1: Phase relations of forsterite-enstatite-$SiO_2$ binary systems at 1.5 and 2.5 GPa, respectively.

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## Question 13 Information Sciences (1)

There are 11 pairs of technical terms below. Choose 4 out of the 11 pairs. For each selected pair, define both of the terms such that the differences between the two terms are clear. Each answer should be approximately 5 lines long.

- deterministic finite state automaton and nondeterministic finite state automaton
- lexical analysis in compilers and parsing in compilers
- propositional logic and first order predicate logic
- message passing based parallel program and shared memory parallel program
- algebraic curve and parametric curve
- scanline method and Z-buffer method
- CPU and GPU
- dynamic programming and greedy algorithm
- Prim's algorithm and Kruskal's algorithm
- UDP (network protocol) and TCP (network protocol)
- maximum flow (graph algorithm) and minimum cost flow (graph algorithm)

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## Question 14 Information Sciences (2)

Read the following text and answer the accompanying questions 1-4.

Rasterization is the process of converting a vector representation of an image to a set of discrete pixels so that the image can be displayed on a display monitor.

Consider rasterization of a 2D line segment with endpoints A $(x_A, y_A)$, B $(x_B, y_B)$. The line containing this segment is defined as follows:

$$Y = \frac{\Delta y}{\Delta x}(X - x_A) + y_A \quad (1)$$

where $\Delta x = x_B - x_A$, $\Delta y = y_B - y_A$

Assume $x_A, y_A, x_B, y_B$ are all non-negative integers, $x_A < x_B$, and $0 \leq |\Delta y/\Delta x| < 1$. When rasterizing such a line segment, we consider each integer value in the $X$ direction, compute an integer value for the $Y$ direction by rounding, and then plot the corresponding pixel.

1. Suppose there is a line segment with endpoints A (2,3), B (8,6). Enumerate the (integer) $x, y$ coordinates of all pixels that are plotted when the line segment in equation (1) is rasterized.

2. The basic idea behind rasterization is to choose pixels to plot based on the vertical distances between the line segment in equation (1) and the centers of each candidate pixel. The pixel closest to the line with respect to the vertical distance is plotted. For example, in **Figure 1**, let $f(x, y)$ be the difference in the $Y$ direction between the center of pixel $(x, y)$ and the line segment at $X = x$. The pixel to be plotted is determined based on $|f(x, y)|$.

   Express $f(x + 1, y)$ in terms of $f(x, y)$.

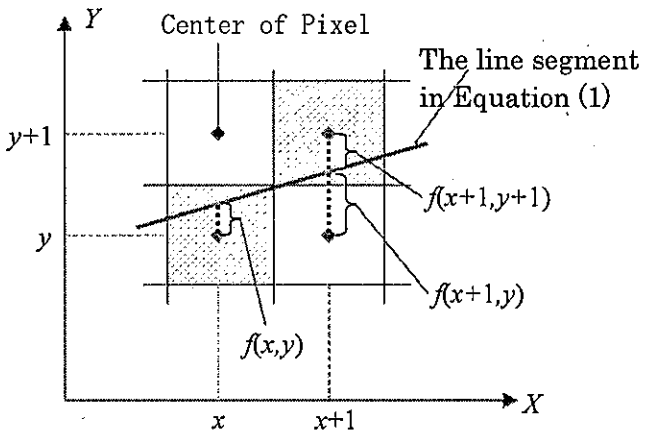


**Figure 1**

3. Consider a program which rasterizes the line segment as $x$ increases from $x_A$ to $x_B$. Note that when $x$ is incremented, the next pixel's $y$ coordinate can either increase by 1 or remain unchanged.

   For efficiency, the following incremental method can be used. Use a variable $e$ such that the relation $e = f(x, y)$ holds. As $x$ is incremented, $e$ can be incrementally updated, where the new value of $e$ depends on whether $f(x + 1, y)$ (from sub-problem 2 above) is less than 0.5.

   Based on these ideas, give an algorithm for rasterization. Your solution can be a program fragment (function, method, block), and does not need to be a complete program. If you use a programming language or notation other than C, C++, Java, Perl, Python, or Ruby, explain your notation so that the meaning of your solution is clear. Assume that you are provided with a procedure for plotting pixel $(x, y)$.

4. The algorithm outlined above in sub-problem 3 requires adding/subtracting floating point numbers. Improve the efficiency of this algorithm by modifying it so that only integer operations are necessary. The modifications can be presented as either a new algorithm or a textual description of the modification to the original algorithm.

Question 15 Science and Technology Studies (1)

Discuss the impacts of environmental technologies on the structure of society, referring to examples of such technologies.

Question 16 Science and Technology Studies (2)

Discuss the rise of the Enlightenment as a movement in the 17th and 18th centuries from the viewpoint of the history and philosophy of science. What were the purposes and motivations of this new movement? What were the reactions of religious, social, and political institutions to this movement? What were its influences upon the development of science?

## Question 17 Economics (1)

Explain briefly the meanings and/or concepts of the following technical terms.
(Graphical explanation is also allowed if it is necessary.)

(1) Indifference curves

(2) First, second, and third-degree price discriminations

(3) The Edgeworth box diagram and the contract curve

(4) Externalities

(5) The prisoners' dilemma

(6) Total factor productivity

(7) The permanent-income hypothesis

(8) Money supply

(9) Expected utility

(10) The capital asset pricing model (CAPM)

## Question 18 Economics (2)

Consider a firm that produces a single commodity. The firm's production function is written as $F(\mathbf{x})$ where $\mathbf{x} \equiv (x_1,\ldots,x_N) \in \Re^N$ denotes the vector of the inputs. Let $\mathbf{p} \equiv (p_1,\ldots,p_N) \in \Re^N$ denote the vector of input prices. Also, let $y$ denote the output level. The firm's cost function is described as follows:

$$C(\mathbf{p},y) \equiv \min_{\mathbf{x}\in\Omega(y)} \mathbf{p}\cdot\mathbf{x} \quad \text{where} \quad \Omega(y) \equiv \left\{\mathbf{x} \in \Re^N \middle| F(\mathbf{x}) \geq y\right\}.$$

(1) Show that $\dfrac{\partial C(\mathbf{p},y)}{\partial p_j} = x_j^*(\mathbf{p},y)$, $j=1,\ldots,N$ where $\mathbf{x}^*(\mathbf{p},y) \equiv \left(x_1^*(\mathbf{p},y),\ldots,x_N^*(\mathbf{p},y)\right)$ represents the solution vector for the above cost minimization problem.

(2) Show that the cost function $C(\mathbf{p},y)$ is homogeneous of degree 1 with respect to $\mathbf{p}$.

(3) Assume that the production function $F(\mathbf{x})$ exhibits constant returns to scale. With this assumption, show that the cost function $C(\mathbf{p},y)$ is homogeneous of degree 1 with respect to $y$.

(4) Consider a special case of $N=2$. Assume that the production function has a Cobb-Douglas form, that is:

$$F(x_1,x_2) = Ax_1^{\alpha}x_2^{1-\alpha}.$$

Show that the cost function is represented by the following:

$$C(p_1,p_2,y) = \left(\frac{p_1}{\alpha}\right)^{\alpha}\left(\frac{p_2}{1-\alpha}\right)^{1-\alpha}\frac{y}{A}.$$

(5) In the case of $N=2$, assume instead that the production function has a CES form, that is:

$$F(x_1,x_2) = A\cdot\left\{\alpha x_1^{\rho} + (1-\alpha)x_2^{\rho}\right\}^{\frac{1}{\rho}}.$$

Show that the cost function is represented by the following:

$$C(p_1,p_2,y) = \left\{\alpha\left(\frac{p_1}{\alpha}\right)^{\sigma} + (1-\alpha)\left(\frac{p_2}{1-\alpha}\right)^{\sigma}\right\}^{\frac{1}{\sigma}}\frac{y}{A} \quad \text{where} \quad \sigma \equiv \frac{\rho}{\rho-1}.$$

# Scrap paper

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